This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

TREATISE
ON THE ART OF CONSTRUCTING
ARCHES
WITH SPIRAL COURSES.

BY W. DONALDSON, M. A. CANT.

A TREATISE

ON

THE ART OF CONSTRUCTING OBLIQUE ARCHES WITH SPIRAL COURSES.

BY

WILLIAM DONALDSON, M.A. CANTAB.,
ASSOC. INST. C.E.

BIBLIOTHECA·BODLEIANA

LONDON:
E. & F. N. SPON, 48, CHARING CROSS.
1867.

186. e. 19.

LONDON:
PRINTED BY C. WHITING, BEAUFORT HOUSE, STRAND.

PREFACE.

THE author's aim has been to arrive at a series of formulæ which would give the data necessary for the construction of oblique arches, without having recourse to any developments on a large scale, or which require skill in draughtmanship.

In the three instances in which recourse is had to scaling, the methods of obtaining the results by calculation are pointed out, but they are very complicated, whereas the development and elevations required are very simple. A knowledge of logarithms only will suffice for the practical application of the formulæ.

A TREATISE, &c.

CHAPTER I.

PROPERTIES OF SCREW SURFACES.

I. SUPPOSE A B an axis, and M N a radius perpendicular to it; if an uniform motion of rotation around and translation along A B be given to M N, it will sweep out a screw surface.

Let A B (Fig. 1) be taken for axis of z, A C perpendicular to it for axis of x, and A D perpendicular to the plane C A B for axis of y.

Let A C be the initial position of the revolving radius M N.

$x\,y\,z$, co-ordinates of any point N in it at time t.

$\alpha\,\beta$, velocities of translation and rotation respectively.

Then, if the rotation take place towards the positive part, A D, of the axis of y, we shall have

$$z=\alpha\, t$$
$$x=r \cos.\,\beta\, t$$
$$y=r \sin.\,\beta\, t,$$

where $r=\sqrt{x^2+y^2}$ = distance of the point $(x\,y\,z)$ from the axis of z; from these we may deduce one equation, that to the screw surface swept out by M N

$$z=\frac{\alpha}{\beta}\tan.^{-1}\frac{y}{x}=c\,\theta \quad (1)$$

where θ is equal to the angle between the plane passing through the axis of z and the revolving radius, and the initial plane C A B, and c is equal to the velocity of translation divided by the angular velocity.

Def. "The space due to the velocity of translation whilst the radius makes half a revolution, is called the axial length of the screw surface," whence we see that c is equal to the axial length divided by π.

II. Let the screw surface $z = c\,\theta$ (Fig. 2) be intersected by a cylinder having the same axis, of radius r, whose length A D is equal to the axial length of the screw surface; let A S C be their line of intersection, B M N C the development of the semicylinder, and C M that of the screw line A C; take any point S in A C, and draw the ordinary S P parallel to the axis meeting D C in P; let S′ P′ be the development of S P, then we have

$$\frac{S'\,P'}{P'\,C} = \frac{S\,P}{P\,C} = \frac{c\,\theta}{r\,\theta} = \text{a constant};$$

and therefore C M is a straight line.

Similarly, and by the converse of the above, if a straight line B Q be drawn cutting C M at any angle, it will be the development of a screw line, B E, of axial length, E A = M Q, which cuts the screw line A S C at the same angle.

III. Let two concentric cylinders, whose radii are r and $r + e$, be cut by a series of similar screw surfaces, described round the axis of the cylinders by radii whose least distances, when in the same plane, are equal to one another: the solid contained between the two cylinders will be divided into a series of similar layers: if, moreover, these layers be again cut by a series of similar screw surfaces, the least distances of whose describing radii, when in the same plane, are likewise equal to one another, then the solid contained between the two cylinders will be divided into a number of equal and similar solids, bounded by the two cylinders and the screw surfaces: if the axial length of the second series of screw surfaces be such, that their traces on any cylinder, whose radius lies between r and $r+e$, cut those of the other screw surfaces on the same cylinder at right angles, these equal and similar solids will be voussoirs of an oblique bridge. The intradosal and extradosal lines of the faces of such arches are usually ellipses in one plane: the method of making bridges with screw faces will first be given; afterwards one in which the bounding lines are ellipses, but not in one plane—the surface contained between them will be a twisted surface, generated by the revolving radius, the velocities of translation and rotation not being in this case uniform; and, finally, the method of making them with elliptic bounding lines in one plane.

IV. In making the calculations necessary for the construction of the templates, it will be best to take the general case, viz., that of a segmental arch, because it includes the case of a semicircular arch. Moreover, as will be seen in Chapter II., an oblique arch ought always to be segmental.

Def. "The angle of obliquity, in an oblique bridge, is the angle between a plane perpendicular to the axis of the cylinder and the plane of direction of the bridge."

Def. "The angle contained between the development of a joint-bed screw line on the soffit or extrado, and a line parallel to the axis of the cylinder, is called the angle of the skewback of the intrado or extrado respectively; or, more generally, the angle contained by the development of any screw line and a line parallel to the axis is the skewback angle of that screw line."

The data are the direct span, the angle of obliquity, the versed sine of the arch, and the width of the pathway. The quantities we have to find are the axial lengths of the two series of screw surfaces, and various soffit dimensions, such as the length of a facial and joint screw line, the breadth and length of any voussoir, measured along the bounding screw lines, and also parallelly and perpendicularly to the axis, the length of the springing, &c.

V. In direct bridges the soffit face is divided into an odd number of voussoirs, in order that there may be a keystone at the centre of the arch; this may always be secured in one of the faces of an oblique arch, but not necessarily in the other.

Let A B C D (Fig. 3) be the development of the soffit of an oblique arch; A D, B C being the developments of the facial screw lines, and A M, N C of the soffit joints, which pass through the extremities A, C, of the springing. Suppose A D divided exactly into an odd number of voussoirs, so that the face A D has a keystone at its centre; if then B M, M C each contain an integral number of these voussoirs, or, in other words, if a joint line pass through the extremity C, then the face B C will also have a central keystone. The following method will show when this is practically possible: let a, b, be the lengths into which the facial screw line is divided by the joint A M, so that

$$BM=a, \qquad MC=b;$$

if m, n be the number of voussoirs in B M, M C, then we must have

$$\frac{a}{m}=\frac{b}{n} \quad (1);$$

we must likewise have $m+n$ an odd number. Since the part between the joint A M and the springing A B is exactly similar to the part between the joint line N C and the springing C D, the voussoir joints in the bed

A M will correspond with the voussoir joints in the bed N C, it will therefore be necessary that n should be an odd number, in order that no cross joints of contiguous layers should be in the same line. Let the fraction $\frac{a}{b}$ be reduced to its lowest terms; when this is done, the numerical values of the numerator and denominator, which express the least number of layers into which A M, M C can be divided, may be too large for practical purposes, or if small enough $m+n$ may be an even integer. In these cases it will be impossible to have a keystone in each face without altering the thrust of the bridge; if it be thought necessary to do so, we must change the axial length of the voussoir beds. The having a keystone, however, for each face of an oblique bridge can add nothing to its strength. If a section were made exactly midway between the two faces, the middle point of that section might be the joint of a bed, or the middle point of a voussoir, or any point between the middle of a voussoir and a joint; since the heaviest pressure comes about the centre of the bridge, it would seem most natural to give the bridge a keystone at the centre; all, therefore, that is necessary is, that n should be an odd number. If the equation $\frac{m}{n} = \frac{a}{b}$ do not give practicable values for m and n subject to this condition, we must divide the length b into any odd number of voussoirs we may think proper; portion a will then contain a certain number of the same thickness, and one upon the springing, of a less or greater thickness, according as we determine to make a separate voussoir of the excess, or to add it to the last full-sized voussoir; the condition, n being an odd number, might of course be got rid of, by breaking joint in the layers of the similar portions on the springing in a different manner; but this would involve a deal of additional trouble.

VI.

Let s=the direct span of the bridge.
h=the versed sine of the arch.
p=width of the pathway.
δ=the angle of obliquity.
e=the depth of a voussoir.

These are the data; let the following symbols denote the quantities to be found:

ω=angle subtended at the centre by the arc.
α=angle of skewback of facial soffit screw-line.
β= „ „ joint soffit „
r=radius of the soffit cylinder.
l=axial length of the joint beds.
λ= „ of the facial screw-line.
t=thickness of a voussoir.

Let a, b have the same meanings as before. It is not necessary to give symbols to any more of the quantities which we have to find.

Let A B C D (Fig. 4) be the projection of the segmental cylinder of the soffit: B C, P B being the same fractions of the axial lengths of the joint and facial screw lines respectively that the arc of the segment is of the semicircumference; take A O, P Q each equal to the length of the springing; join O Q, A P, then O Q, A P are the projections of the soffit ellipses on a plane passing through the springings; let the curved lines joining A P and O Q be the projections of the facial screw lines; draw P P′ perpendicular to O Q; then P P′ is the direct soffit width of the bridge; let B N M C be the development of the soffit cylinder, the straight lines P N, Q N′ of the screw lines, and the curved lines P N, Q N′ of the ellipse lines of the soffit face. The developments of the soffit joint lines will be parallel to P M, and

$$\text{radius of soffit cylinder } r=\frac{s^2+4h^2}{8h}$$

$$\text{angle subtended by the arc } \omega=2\sin.^{-1}\frac{4hs}{s^2+4h^2}.$$

When the bridge has plane faces, the angle B A P is the angle of obliquity of the bridge; when the faces are screw surfaces, the angle of obliquity is the angle contained by the plane of the extradosal ellipse and a plane perpendicular to the axis; the method of finding the angle B A P, when this is the case, will be given in the next article: call it δ', then we shall have

$$\begin{aligned}\text{the length of springing} &= \text{P P}' \sec. \text{P}' \text{P Q}\\ &= \text{PP}' \sec. \delta'\\ \text{and this must be} &= p \sec. \delta;\end{aligned}$$

also axial length of facial screw lines: B P : : semicircumference : arc of soffit segment:

$$\lambda : s \tan. \delta' :: \pi r : \omega r$$

$$\therefore \text{ axial length } \lambda = \frac{\pi \tan. \delta'}{\omega} s.$$

In order to find the axial length of the joint beds, it is necessary to know the angle of the skewback of that facial screw line which cuts the corresponding screw lines of the joint beds at right angles: let ρ be the radius of the cylinder on which these lines lie, so that ρ lies between r and $r + e$; now the angle B P N is the skewback angle of the soffit facial screw line; if BMNC were the development of the cylindrical segment radius ρ, the angle B P N would be the skewback angle we are seeking: hence the cotangent of this angle is equal to B P, divided by the length of the corresponding arc, since B P is constant for each of them: hence

$$\text{the cotangent} = \frac{s \tan. \delta'}{\omega \rho}.$$

Also the angle corresponding to N P R* being a right angle, the angle corresponding to Q P R, the skewback angle required, is the complement of the angle just found. Hence its tangent is equal to $\frac{s \tan. \delta'}{\rho \omega}$, and therefore the axial length of the joint beds

$$l = \frac{\pi \omega \rho^2}{s \tan. \delta'}.$$

---

* P R R is a soffit joint line, meeting facial screw and ellipse lines in R R′ respectively.

and therefore

$$\text{tan. }\beta=\frac{\text{arc of soffit semicircle}}{\text{axial length of joint beds}}$$

$$=\frac{r\,s\,\text{tan. }\delta'}{\rho^2\,\omega}$$

$$\text{of course tan. }\alpha=\frac{\omega\,r}{s\,\text{tan. }\delta'}.$$

$$\text{length of facial screw line }a+b=\text{P N}=\text{B P sec. }\alpha$$
$$=s\text{ tan. }\delta'\text{ sec. }\alpha$$

$$\text{length of portion Q R, or }a=\text{Q P}\,\frac{\text{sin. Q P R}}{\text{sin. Q R P}}$$

$$=\frac{p\text{ sec. }\delta\text{ sin. }\beta}{\text{sin. }(\alpha+\beta}$$

length of a soffit screw line contained between two facial soffit lines,

$$\text{P R}=\text{Q P}\,\frac{\text{sin. P Q R}}{\text{sin. Q R P}}=\frac{p\text{ sec. }\delta\text{ sin. }\alpha}{\text{sin. }(\alpha+\beta)}.$$

From these values of a and b and P R, we can determine the proper thickness and length of a voussoir. This being done, we shall have length of springing intercepted between contiguous soffit joint lines.

$$=\text{thickness}\times\frac{\text{sin. Q R P}}{\text{sin. Q P R}}$$

$$=t\,\frac{\text{sin. }(\alpha+\beta)}{\text{sin. }\beta}.$$

Similarly, length of the circular arc, intercepted between two contiguous joint lines, is

$$=t\,\frac{\text{sin. }(\alpha+\beta)}{\text{cos. }\beta};$$

and length of projection of voussoir thickness on a circular arc = $t \sin \alpha$, horizontal width of springing, measured perpendicularly to the face of the abutment, is $e \sin \frac{\omega}{2}$, and measured obliquely is $e \sin \frac{\omega}{2} \sec. \delta$.

VII. In the case of arches with twisted faces, the quoins of the springing will be on the square; when the bounding lines are ellipses, the plane of the extradosal ellipse will be in the direction of the roadway, and therefore the angle of obliquity will be the angle contained by this plane and one perpendicular to the axis.

Let A B C D, A′ B′ C′ D′, Fig. 5, be the ground plans of the abutments, join A A′, B B′: produce A B till it meets C′ B′, D′ A′, in F and E. Then we have

$$\delta = A'AE$$
$$\delta' = B'BF.$$

$$\text{Now } B'F = A'E = AE \tan. A'AE$$

$$= 2(r+e)\sin.\frac{\omega}{2}\tan.\delta,$$

$$\text{and therefore } \tan.\delta' = \frac{2(r+e)\sin.\frac{\omega}{2}\tan.\delta}{s} = \frac{r+e}{r}\tan.\delta.$$

In the case of bridges with twisted faces, it would seem best to make the screw lines on the extradosal cylinder cut each other at right angles.

If oblique brick arches were built with twisted faces and bounding lines ellipses, it would be necessary neither to cut the bricks nor to leave an unworkmanlike toothing: no more lines would be required on the laggings than for one turned in the ordinary manner, and very much less care would be required on the part of the bricklayer, since, instead of having to keep the faces plumb, by making the last course of face bricks flush with the preceding, the joints would of themselves keep the proper radiation.

VIII. Before giving the method of determining the lengths of the soffit lines, when the bounding facial lines are ellipses, it will be proper to show how far a facial screw line deviates from the plane of the facial ellipses. In cases where this deviation is small in amount, it will not be necessary to take the trouble of making the facial lines ellipses. Moreover, the investigation will show the way of obtaining the lengths required.

In Fig. 4 draw L $u\,v\,u'\,v'$, meeting the developments of the ellipse lines in $v\,v'$, and those of the screw lines in $u\,u'$. Take B for the origin, B N for the axis of x, B C for the axis of y. Let B L be the development of an arc which subtends an angle φ at the centre; then, with the previous notation, we shall have for co-ordinates of v, u, respectively

$$\left.\begin{array}{l} x= r\,\varphi \\ y=\left\{s+2\,r \sin.\left(\frac{\omega}{2}-\varphi'\right)\right\}\frac{\tan.\,\delta}{2} \end{array}\right\}\text{co-ords. of } v \begin{array}{l}(1)\\(2)\end{array}$$

$$\left.\begin{array}{l} x=r\,\varphi \\ y=r\,(\omega-\varphi)\cot.\,\alpha \end{array}\right\}\text{co-ords. of } u.$$

These, increased by the lengths of the springing, are also the co-ordinates of $u'\,v'$ respectively. From these we find

$$u\,v=u'\,v'=\left\{s+2\,r\sin.\left(\frac{\omega}{2}-\varphi\right)\right\}\frac{\tan.\,\delta'}{2}-r\,(\omega-\varphi)\cot.\,\alpha.$$

If $p\,v$, $p'\,v'$ be drawn parallel to the screw lines of the soffit joints, and meeting the soffit facial screw lines in $p\,p'$, we shall have

$$p\,v=p'\,v'=\frac{u\,v\sin.\,\alpha}{\sin.\,(\alpha+\beta)}.$$

In order, therefore, to find the lengths of the soffit joints, we must find the values of $p\,v$, $p'\,v'$, corresponding to every joint line. Suppose the n^{th} joint on one face correspond with the m^{th} joint on the other, we must find the corresponding values of $p\,v$, $p'\,v'$, and add them to the length of the soffit bed between the two facial screw lines, having careful regard to the sign of these differences. To ascertain these differences by calculation would be extremely difficult and laborious, since it would be necessary to find the co-ordinates of the points of intersection of every soffit joint line with the two facial soffit ellipses. By giving successive values to x, in equation (1), we shall find the corresponding values of ϕ in circular measure, and thence the corresponding value in degrees. These being successively substituted for φ, in equation (2), will give the corresponding values of y. If these abscissæ and ordinates be carefully plotted, the lengths $p\,v$, $p'\,v'$, can be obtained with sufficient accuracy by scaling.

IX. When the bounding lines of the faces are ellipses in one plane, the lines of intersection of the voussoir beds with these planes are curves which pass, when prolonged, through the axis of the cylinder; they differ very little, however, in the short length of a voussoir bed from the chord joining their extreme points. These chords all intersect a vertical line through the axis very nearly in the same point, whose distance from the axis may be determined as follows:

Let A C (Fig. 6) be the axis of the cylinder, A D, passing through the axis, and parallel to the line of intersection of the facial plane with any plane perpendicular to the axis. Take A C for the axis of z, A D for that of y, and a line, A B, perpendicular to each of these for axis of x. Let the facial plane intersect the plane of $x\,z$ in the line A E, and the bounding screw lines of a joint bed in the points P P′; let $p\,p'$ be the projections of P P′ on $x\,z$. Draw $p\,q$, $p'\,q'$ perpendicular to A B; now the straight line joining P P′ must meet the axis A D. Let it meet it at a distance, d, from the origin measured negatively, suppose, then we shall have

$$\frac{\mathrm{P}p+d}{\mathrm{P}'p'+d}=\frac{\mathrm{A}\,p}{\mathrm{A}p'}=\frac{\mathrm{A}\,q}{\mathrm{A}\,q'} \quad (1).$$

Adopting the previous notation, if we suppose the radius to have moved a distance h along the axis of z, before it reached the plane B A D we shall have

$$\left.\begin{array}{l} z=c\,\theta-h \\ x=r\cos.\theta \\ y=r\sin.\theta \end{array}\right\} \text{(2) co-ordinates of P.}$$

$$\left.\begin{array}{l} z=c\,\theta'-h \\ x=\overline{r+e}\cos.\theta' \\ y=\overline{r+e}\sin.\theta' \end{array}\right\} \text{(3) co-ordinates of P}';$$

also the equation to the plane D A E is

$$z = x \tan.\delta \quad (4).$$

From equations (1), (2), (3), we obtain

$$\frac{r \sin.\theta + d}{\overline{r+e}\sin.\theta' + d} = \frac{r \cos.\theta}{\overline{r+e}\cos.\theta'}.$$

$$\therefore \quad d = \frac{r.\overline{r+e}\sin.(\theta'-\theta)}{\overline{r+e}\cos.\theta' - r\cos.\theta}$$

Since the points P P′ are in the plane D A E, we have

$$c\,\theta - h = r\cos.\theta\tan.\delta$$

$$c\,\theta' - h = \overline{r+e}\cos.\theta'\tan.\delta$$

$$\therefore \quad \overline{r+e}\cos.\theta' - r\cos.\theta = c\,(\theta'-\theta)\cot.\delta$$

$$\therefore \quad d = \frac{r.\overline{r+e}\sin.(\theta'-\theta}{c\cot.\delta\,(\theta'-\theta)}.$$

Now, $\theta'-\theta$ is always very small,

$$\therefore \; d = \frac{r\,(r+e)}{c\cot.\delta}, \text{ nearly,}$$

which, being independent of θ, is the same for every bed; reducing, we obtain

$$d = \frac{2\,(r+e)^{*}}{\pi\cot.^2\delta}.$$

X. In order to use one of the templates, it is necessary to know the distances of the last-found point from the points of intersection of the soffit ellipse with each soffit joint line, and also from the feet of the describing radii, which pass through these points of intersection. For the reasons before stated, to ascertain these by calculation would be extremely laborious. The first distances may be obtained by scaling from an elevation of the face; the second distances are the hypothenuses of right-angled triangles, one of whose sides is the constant

---

[*] More exactly $d = \frac{2.\overline{r+e}}{\pi\cot.^2\delta}\cdot\frac{r^2}{\rho^2}$; but since ρ lies between r and $r+e$, and e is very small compared with r, $\rho^2 = r^2$ very nearly in all cases.

distance d, and the other the successive distances of the feet of the describing radii above mentioned from the point where the facial plane cuts the axis. These distances may be obtained by scaling from the soffit development. In Fig. 18, C D E is the axis of the cylinder; D, the point where it meets the facial plane; O D is the distance, d; the points marked 1st, 2nd, 3rd, &c., show the distances of the feet of the radii passing through the 1st, 2nd, 3rd, &c., joints from each springing from the point D. The length of the hypothenuse is at once obtained by scaling.

XI. In Fig. 7, let O z, the axis of the cylinder, be taken for axis of z; O x, O y, at right angles to it and to each other, for axes of x and y; let C D E B be the bed of a voussoir swept out by the revolving radius A B in moving through an angle, θ, from its initial position, O x; let O C′ B′ be the projection of A C B, C′ B′ being the projections of C B respectively; join D C′, E B′, then $OD = OC' = r$, $OE = OB' = r + e$; since the angle θ is very small, the chords D C′, E B′, may be taken for the arcs; now these arcs are the same fractions of the semicircumferences of the soffit and extradosal cylinders that C C′ is of the axial length of the screw surface. Since C C′ is parallel to the axis, the angle C′ C D is equal to the skewback angle of the soffit joint lines, β; through B draw B p parallel to C D; the screw lines C D, B E, may be supposed to coincide with the straight lines joining C D and B E, and through E draw E q perpendicular to B p, produced, meeting it in q. Now, by construction, C′ C, C D, are respectively parallel to B′ B, B p; therefore $\angle B'Bp = \angle C'CD = \beta$; also q E, p E, are respectively perpendicular to B p, B B′, therefore $\angle pEq = \angle B'Bp = \beta$. Now

$$pB' = C'D = K \sin. \beta, \text{ if length } CD = K$$

$$pE = EB' - pB' = \frac{r+e}{r} CD' - CD'$$

$$=\frac{e\,K}{r}\sin.\beta$$

$$q\,E=p\,E\cos.\beta=\frac{e\,K}{r}\sin.\beta\cos.\beta$$

$$p\,q=p\,E\sin.\beta=\frac{e\,K}{r}\sin^2\beta.$$

XII. It is impossible to make the joint beds perfect screw surfaces. In making them two chisel drafts only are generally sunk in the bed, and the rest of the surface dressed off until a straight edge, held as nearly as possible perpendicular to each of these drafts, will coincide with every part of the bed as it is moved along them. The amount of error entailed by this mode of dressing them may be shown as follows:

Let A D (Fig. 8) represent the axis of the screw surfaces; take it for axis of z, and A B, A C, at right angles to it and to each other, as axes of $x\,y$; let A O, M′ M, N′ N be positions of the generating radius such that A M′ is equal to M′ N′; if M M′ = r, and θ be the angle through which the radius has revolved in moving from the position A B, the equation to a plane through M perpendicular to M′ M will be

$$x\cos.\theta+y\sin.\theta-r=0 \quad (1).$$

The plane of the straight edge is supposed to be always held parallel to this plane; let N be the point where it meets the radius N N′; the projections of N M on the plane of $x\ y$ will lie in the line of intersection of that plane with the plane represented by (1); let $n\,m$ be the points of projection; then since $\angle$ O A m = $\angle$ n A m = $\angle$ θ, and the $\angle$ O m A = $\angle$ A $m\,n$, by construction we have

$$n\,m=m\,\mathrm{O}:$$

$$\text{but } \mathrm{N}\,n=2\,\mathrm{M}\,m;$$

and therefore the three points N, M, O, lie in the same straight line (α). Suppose the radial line P p' meet the plane (1) in p', let $\mu\,\theta$ be the angle through which it has revolved; if A q be its projection, we shall have $p'\,q = c\,\mu\,\theta$, where c is some constant; if p be the correspondent point in the straight line N, M, O, we shall have

$$\frac{p\,q}{\mathrm{M}\,m} = \frac{r\ \tan.\,\overline{\mu-1}\,\theta + r\tan.\,\theta}{r\tan.\,\theta},$$

$$\text{or } p\,q = c\,\theta + c\,\theta\,\frac{\tan.\,\overline{\mu-1}\,\theta}{\tan.\,\theta};$$

and therefore the difference between these two, which bears a constant ratio to the error estimated perpendicularly to the voussoir bed, is

$$p'\,q - p\,q = c\,\mu\,\theta - c\,\theta - c\,\theta\,\frac{\tan.\,\overline{\mu-1}\,\theta}{\tan.\,\theta}$$

$$= c\,\theta\left(\overline{\mu-1} - \frac{\tan.\,\overline{\mu-1}\,\theta}{\tan.\,\theta}\right) \quad (\beta),$$

from the results (α) and (β), we see that (1) there are but three lines in each voussoir bed which coincide with the screw surface; (2) that the error is inappreciable when θ is so small that $\theta = \tan.\,\theta$ nearly.

CHAPTER II.

STABILITY OF OBLIQUE ARCHES.

The following investigation into the stability of oblique bridges is based on these two assumptions:

I. That the material can resist any pressure that may come upon it.

II. That, for perfect security, no force of shearing must be exerted on any of the voussoirs.

The forces acting on a voussoir are its own weight, a portion of the weight passing over the bridge, and the pressures on its beds. No force is exerted on the ends of a voussoir; in oblique bridges it is kept in its place by the friction on the beds. If there were no backing, these forces must be in equilibrium: this is never the case where the live load is variable; they must, therefore, have a definite resultant. We may resolve these pressures in three directions—vertically, parallelly to the axis, and perpendicularly to these two; the resultant of the vertical components will be equal and opposite to the weight of the voussoir and of the load upon it, and, in order that the horizontal components may satisfy condition II., their resultants must fall within the springing nearest to the bed to which they refer, so as to meet the material with which the bridge is backed up; also, since the pressures between the voussoir beds are mutual, we see that the horizontal resultant of either pair must pass within each springing.

III. Now the pressure at any point of a voussoir bed will act in the direction of a normal to the surface at that point, and will therefore lie in the normal plane to the screw line passing through that point, and will also be perpendicular to the revolving radius; and therefore its line of direction will be the line of intersection of the above normal plane and a plane through that point perpendicular to the radius: we may assume the pressure at every point along this radius to be the same.

IV. Take O B (Fig. 9), the axis of the cylinder, for axis of z; O C, in a horizontal plane, passing through the axis of z, and perpendicular to that axis, for axis of x; and O A, perpendicular to these last, for axis of y, the plane ($y\,z$) will divide the bridge symmetrically. Take $x\,z$ for the initial plane, and suppose the radius to revolve in direction of the arrow.

Let Q P be a screw line in the bed of a voussoir, ρ the radius of its cylinder, and θ the angular distance of P from the initial plane; let l be the axial length of the screw surface, and h the distance of Q, where the screw line meets the initial plane from the axis of x, then the co-ordinates of the point P will be

$$z=c\,\theta+h,\ \text{where } c=\frac{l}{\pi}$$

$$x=\rho\cos.\theta$$

$$y=\rho\sin.\theta.$$

Hence the equation to the normal plane to the screw line at the point x, y, z will be

$$-(X-x)\,\rho\sin.\theta+(Y-y)\,\rho\cos.\theta+(Z-z)\,c=0 \quad (1),$$

and the equation to a plane through the same point perpendicular to the radius is

$$x\cos.\theta+y\sin.\theta-\rho=0 \quad (2).$$

If α, β, γ, be the angles of inclination of the line of intersection of these two planes to the axes of x, y, z, respectively, we shall have

$$\cos.\ \alpha = \frac{c \sin.\ \theta}{\sqrt{c^2+\rho^2}}$$

$$\cos.\ \beta = -\frac{c \cos.\ \theta}{\sqrt{c^2+\rho^2}}$$

$$\cos.\ r = \frac{\rho}{\sqrt{c^2+\rho^2}}.$$

if, therefore, $p\ \delta\rho\ \delta s$ be the pressure on the element $\delta\rho\ \delta s$, since this pressure is supposed to be the same at every point along the radius, we shall obtain for the compenents of the resultant of the pressures which act along the radius :

$$\mathrm{P}x = \Sigma\ \frac{p\, c \sin.\ \theta}{\sqrt{c^2+\rho^2}}\,\delta s\, \delta\rho = p\, c \sin.\ \theta\ \delta s \int_r^{r+e} \frac{d\rho}{\sqrt{c^2+\rho^2}}$$

$$= p\, c \sin.\ \theta\ \delta s \log.\ \frac{r+e+\sqrt{c^2+\overline{r+e}^2}}{r_{\!,}+\sqrt{c^2+r^2}}$$

$$= p\, c \sin.\ \theta \log.\ u\ \delta s.$$

Similarly

$$\mathrm{P}_y = -p\, c \log.\ u \cos.\ \theta\ \delta s$$

$$\mathrm{P}z = \Sigma\ \frac{p\,\rho}{\sqrt{c^2+\rho^2}}\,\delta s\, \delta\rho = p\left(\sqrt{c^2+\overline{r+e}^2} - \sqrt{c^2+r^2}\right)\delta s$$

$$= p\, v\ \delta s,$$

where $u\ v$ are constant for every position of the radius, and r, $r+e$ are the radii of the soffit and extradosal cylinders.

Let X Y Z be the components of the resultant pressure upon the whole voussoir bed, $\alpha\ \beta$ values of θ corresponding to the ends of the bed, then we shall have, if we suppose the pressure at every point in the bed the same,

$$\mathrm{X} = \int_r^{r+e} \int_\beta^\alpha \frac{p\, c \sin.\ \theta}{\sqrt{c^2+\rho^2}}\, d\rho\, d s,$$

$$Y = -\int_{r}^{r+e} \int_{\beta}^{\alpha} \frac{p\, c \cos.\, \theta}{\sqrt{c^2+\rho^2}}\, d\rho\, ds,$$

$$Z = \int_{r}^{r+e} \int_{\beta}^{\alpha} \frac{p\, \rho}{\sqrt{c^2+\rho^2}}\, d\rho\, ds,$$

where

$$ds = d\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2 + r^2 \sin.^2\theta \left(\frac{d\varphi}{d\theta}\right)^2}.$$

The r in this value of θ is the radius vector from the origin, and φ its inclination to the axis of z; therefore

$$r^2 = \rho^2 + c^2\,\theta^2$$
$$r = c\,\theta \sec.\, \varphi$$
$$\rho = c\,\theta \tan.\, \varphi.$$

By means of these, we can express $\frac{dr}{d\theta}, \frac{d\varphi}{d\theta}$ in terms of the independent variables ρ and θ; the integral so obtained is of a very complex form; we require, however, only the values of Px, Py, Pz, to explain the nature of the stability of oblique arches.

As to the magnitude of the forces, since the value of p remains the same for every position of the radius, to which it refers, at the same distance from the centre of the arch; the values of Px, Py, Pz will be the same for the same value of θ, or for all positions of the radius, in any, the same, plane through the axis. Now, in order to find the direction of the resultant, only the relative values of Px, Py, Pz are required; since p is involved as a simple factor in each of them, it will disappear. In semicircular arches, whatever be the value of p, Px vanishes at the springing; thus in these arches condition (II.) can never be fulfilled.

Let A B C D, A′ B′ C′ D′ (Fig. 10) be the ground plan of the springings of an oblique arch with plane faces; Oz, x' Ox, the projections of the axes of z and x; the axis of z being the axis of the soffit cylinder, and the axis of x being in a vertical plane, passing through the extremity, A, of the springing; let P Q be the direction of the resultant of the horizontal components Px, Py, at the point $\left(r+\frac{e}{2}, \theta\right)$ where e is the depth of a voussoir; for the resultant will manifestly pass through the middle point of the depth e, since the pressure is supposed to be the same at every point along it. Now we have shown that the direction of this resultant must pass within both springings; if, therefore, P P$'$ be the line of intersection of the cylinder $r+\frac{e}{2}$ with a plane through the axis, inclined at an angle θ to the initial plane, the direction of the resultant at every intermediate point between P and P$'$ will pass within each springing, provided that the directions of the resultants at P and P$'$ pass through A and A$'$ respectively. Let the soffit arc subtend an angle ω at the centre, then we shall have

$$\mathrm{O\,A}=\frac{s}{2}+e\sin.\frac{\omega}{2};$$

also the tangent of the angle of inclination of P Q to the axis of x is

$$=\frac{\mathrm{P}_2}{\mathrm{P}x}=\frac{v}{c\sin.\theta\log.u};$$

and therefore the equation to P Q A is

$$\mathrm{Z}=\frac{\left(x+\frac{s}{2}+e\sin.\frac{\omega}{2}\right)v}{c\sin.\theta\log.u} \quad (1).$$

Similarly the equation Q$'$ P$'$ A$'$ is

$$\mathrm{Z}=\frac{\left(x-\frac{s}{2}-e\sin.\frac{\omega}{2}\right)v}{c\sin.\theta\log.u}+p\sec.\delta+\left(s+2e\sin.\frac{\omega}{2}\right)\tan.\delta \quad (2);$$

and therefore the width of bridge which satisfies condition (II.) corresponding to the angles θ and $\pi - \theta$ is measured parallel to the axis,

$$p \sec. \delta + \left(s + 2\, e \sin. \frac{\omega}{2}\right) \left(\tan. \delta - \frac{v}{c \sin. \theta \log. u}\right).$$

One of the maximum values of this expression is when $\theta = \frac{\pi}{2}$, or at the centre of the arch; if the arch be semicircular, so that θ vanishes at the springing, we see that for the value $\theta = 0$ this vanishes also; moreover, its value increases as the axial length increases, and this latter increases with the radius of the cylinder, and therefore a segmental arch has a greater width of safety at its centre than a semicircular one of the same direct span and obliquity.

From the above arise at once the two following propositions:

1. Given the obliquity span and angle of arc to find the width of the bridge, in order that there may be a safe pathway of so many feet at the centre.
2. Given the obliquity, span, and width of bridge to find the angle of arc, in order that there may be a safe pathway of so many feet in the centre.

The first is very easily solved; the second involves expressions of the form $\frac{\log. (f(r))}{f(r)}$, and is therefore extremely difficult. The versed sine of the arc should, however, always be as small as possible.

Since the direction of the line of thrust is at no point parallel to the direction of the bridge, it is manifest that in large bridges the wings adjoining the points A A′ should be very much heavier than at D D′.

By a safe pathway of so many feet, it must be understood that the thrust of the bridge, occasioned by weights within those limits, does not violate condition (II.).

CHAPTER III.

FORMATION OF THE TEMPLATES.

In this chapter all the formulæ previously proved are collected together, and their use explained by working out the details of a bridge. An angle of 45° has been chosen, because an instance in which the skew is very great will give the best illustration of the errors introduced by the approximate methods used.

From the previous chapters we have—

Radius of soffit cylinder γ . . . . . $=\dfrac{s^2+4h^2}{8\,h}$

Angle subtended by the arc ω . . . . . $=2\ \text{sin.}^{-1}\dfrac{4hs}{s^2+4\,h^2}$

Length of springing . . . . . . $=p\ \text{sec.}\ \delta$

Axial length of facial screw line, λ . . $=\dfrac{\pi\ \text{tan.}\ \delta'}{\omega}.s$

Do. do. of joint beds, l . . . . $=\dfrac{\pi\ \omega\ \rho^2}{s\ \text{tan.}\ \delta'}$

Angle of skewback of facial screw line, α . $=\text{tan.}^{-1}\dfrac{\omega\ r}{s\ \text{tan.}\ \delta'}$

Do. do. of joint line, β . . $=\text{tan.}^{-1}\dfrac{r\ s\ \text{tan.}\ \delta'}{\omega\ \rho^2}$

Length of facial screw line, or $a+b$ . . $=s\ \text{tan.}\ \delta'\ \text{sec.}\ \alpha$

Length of a . . . . $=\dfrac{p\ \text{sec.}\ \delta\ \text{sin.}\ \beta}{\text{sin.}\ (\alpha+\beta)}$

Length of soffit joint line contained between facial soffit lines . . . . . $=\dfrac{p\ \text{sec.}\ \delta\ \text{sin.}\ \alpha}{\text{sin.}\ (\alpha+\beta)}$

Length of springing intercepted between two contiguous soffit joint lines . . $=\dfrac{t\ \text{sin.}\ (\alpha+\beta)}{\text{sin.}\ \beta}$

Length of circular arc intercepted between two contiguous soffit joint lines . . $=\dfrac{t\ \text{sin.}\ (\alpha+\beta)}{\text{cos.}\ \beta}$

Distance, d, of the points of radiation of the face joints from the axis . . . $=\dfrac{2\,(r+e)}{\pi\ \text{cot.}^2\ \delta}$

Width of springing stones measured at right angles to abutment . . . . $\Big\} = e \sin.\frac{\omega}{2}$

Width of springing stones measured obliquely $= e \sin.\frac{\omega}{2} \sec.\delta$

Length of projection of soffit thickness on circular arc . . . . $= t \sin.\alpha$.

For the formation of one of the templates we shall also require the values of the two fractions :

$$\frac{e}{r} \sin.\beta \cos.\beta \text{ and } \frac{e}{r} \sin.^2\beta.$$

The co-ordinates of the developments of the ellipse lines are given by the equations :

$$x = r\,\varphi.$$

$$y = \left\{ s + 2\,r \sin.\left(\frac{\omega}{2} - \varphi\right) \right\} \frac{\tan.\delta'}{2}.$$

$$'y = p \sec.\delta + \left\{ s + 2\,r \sin.\left(\frac{\omega}{2} - \varphi\right) \right\} \frac{\tan.\delta'}{2}.$$

II. Let these be the data :

Direct span, s . . $= 28'$
Rise of arch, h . . $= 6'$
Direct width, p . . $= 23'$
Angle of obliquity, δ $= 45°$
Depth of voussoirs, e $= 2'$.

Let the faces be ellipses in one plane. This example will explain the construction of the templates in the other two cases. The angle δ is now the angle of obliquity of the soffit. For the sake of illustrating the formulæ, we may make the screw lines on the cylinder, whose radius is $r + \frac{e}{2}$, cut each other at right angles.

Radius, r . . . . $= 19{\cdot}333'$
Angle, ω . . . . $= 92° 47' 47''$
Length of the arc of soffit . . . . . $= 31{\cdot}312'$
Semicircumference do. . . . . . . $= 60{\cdot}737'$
Length of springing . . . . . . $= 32{\cdot}527'$
Axial length of facial screw lines . . . . $= 54{\cdot}312'$
Do. do. joint beds . . . . . $= 75{\cdot}129'$
$\angle$ of skewback of facial screw lines, α . . $= 48° 11' 45''$
Do. do. joint beds, β . . . $= 38° 57' 12''$
Length of facial screw line, $a + b$ . . . $= 42{\cdot}005'$

Length of portion *a*	=20·474′
Length of soffit joint line contained between two facial screw lines	=24·276′
Width of springing stones on the square . .	=1·45′
Do. do. measured obliquely .	=2·05′
Length of oblique span	=39·598′
Distance of the point of radiation of the face joints from the axis of the cylinder . .	=13·555′
Value of fraction $\frac{e}{r}$ sin. β cos. β	=·05057
Do. do. $\frac{e}{r}$ sin.$^2\beta$	=·04088.

If we make the thickness, *t*, 6″, the two parts will contain an exact number of voussoirs—viz. *a*, 41, and *b*, 42. If we make it ·79′, there will be 27 voussoirs in the part *b* and 25 voussoirs and a remainder ·58′ in the part *a*. No allowance is here made for mortar joints: that is a point on which the engineer must exercise his judgment. Adopting the last thickness, we have:

Length of springing intercepted between two contiguous soffit joint lines	=1·2577′
Length of circular arc intercepted between two contiguous soffit joint lines	=1.0173′
Length of projection of soffit thickness or arc .	= ·59013′.

The following table gives the values of ϕ in degrees, and the lengths of the ordinates corresponding to values of x from 1′ to 15′:

	Angle.	Ordinates.	
$x=1'$	2° 57′ 52″	27·291′	59·818
$x=2'$	5° 65′ 44″	26·547′	59·074
$x=3'$	8° 53′ 36″	25·769′	58·296
$x=4'$	11° 51′ 28″	24·960′	57·487
$x=5'$	14° 49′ 20″	24·121′	56·648
$x=6'$	17° 47′ 12″	23·254′	55·781
$x=7'$	20° 45′ 4″	22·363′	54·890
$x=8'$	23° 42′ 56″	21·450′	53·977
$x=9'$	26° 40′ 48″	20·522′	53·049
$x=10'$	29° 38′ 40″	19·571′	52·098
$x=11'$	32° 36′ 32″	18·606′	51·133
$x=12'$	35° 34′ 24″	17·628′	50·255
$x=13'$	38° 32′ 16″	16·643′	49·170
$x=14'$	41° 30′ 8″	15·651′	48·178
$x=15'$	44° 28′ 0″	14·652′	47·179

There may be some who understand the use of tables of logarithms, but do not understand the conversion of an angle in circular measures into one of degrees, minutes, and seconds. Instead, therefore, of putting $x = 1'$, $2'$, $3'$, &c., on the equation $x = r\,\phi$, let ϕ in the value of y be put successively $= 2^\circ$, 4°, 6°, &c.; the corresponding values of x will be

$$\frac{2\,r}{57\cdot296},\ \frac{4\,r}{57\cdot296},\ \frac{6\,r}{57\cdot296},\ \text{\&c.}$$

III. Description of the templates.

In Fig. 11*a*, A B D C is a rectangular piece of sheet iron about $\frac{1}{8}''$ thick, whose length, A C, is equal to the depth of a voussoir, and depth about $2''$. A′ B′ E D′ C′ is another piece of sheet iron of the same thickness, which may be divided into two parts—a rectangle A′ B′ D′ C′ equal in every respect to A B C D, and a right-angled triangle B′ E D′.

Fig. 11*b* shows these two plates connected together by two rectangular plates of the same width and depth, whose lengths A A′, C C′, are such that

$$\text{C C}' = \text{A A}'\ (1 + \frac{e}{r}\sin.^2\beta).$$

They are so connected that the edges A C, A′ C′, in Fig. 11*b*, correspond with the edges A C, A′ C′ in Fig. 11*a*; and in Fig. 11*b* the edges A C, C C′, C′ A′, A′ A are all in the same plane, and the angles A′ A C, A C C′, are right angles. The length A A′ being fixed upon, D′ E, in Fig. 11*a*, is given by the equation

$$\text{D}'\,\text{E} = \text{A A}\,\frac{e}{r}\sin.\beta\cos.\beta.$$

In the figure A A′ has been taken equal to $1\cdot5'$, therefore C C′ $= 1\cdot56'$, and D′ E $= \cdot076'$.

The next template is constructed with more regard to accuracy than those usually adopted for the same object.

In a bridge of great obliquity too much care cannot be used, as success depends entirely upon this template. However, the principles of construction here explained will enable every one to exercise his own judgment.

Fig. 12*a* is the plan of a template whose edges, *g* A, *f* E, *d* D, are circular arcs; the arc *d* D, in Fig. 12*a*, corresponds with the arc *g h k* of the plate *g e f k*, which is about $\frac{1}{8}''$ thick and 3″ deep, in Fig. 12*b*, which is an elevation of the side D *d* of Fig. 12*a*. These circular arcs must be joined together by rectangular plates of equal length, of which A E, E D, B F, F C, Fig. 12*a*, are the edges, in such a manner that the points A, E, D are in the same straight line, and likewise the points B, F, C; also the figures A F, F D must be rectangular, and the lengths of the connecting plates must be such that A D = B C = space moved parallel to the axis by the revolving radius, whilst it describes the arc D H. The connecting plates must be an inch or so less in depth than the others, and must be so attached that their edges next the circular arcs are about one inch distant from those arcs, in order that they may not interfere with the soffit before it is dressed off. F K is a similar plate, put in merely to stiffen the framework. The rods A′, H′, seen in plan in Figure 12*a*, are the same as the rods A′ H′ in Figure 12*b*, seen in elevation. The side B A, in Fig. 12*a*, is produced, in order that the rod A′ may be attached to it. The rods A′, H′, in Fig. 12*b*, are so attached to their respective plates that the edges *f k f′*, *e h e′* are in direction of radii; the lengths, *k f′*, *h e′*, being each equal to the depth of a voussoir, and the arc *k h* equal to the arc swept out by the revolving radius, whilst it moves parallel to the axis through a distance A D. The arcs A *g*, H *d* must not be less than the depth of a voussoir measured along the arc, and the arc E *f* must be rather more than a mean between the length of

A *g*, D *d*. It is manifest that if this template be applied to a voussoir already dressed, so that the edges *k f'*, *h e'* coincide with the bed, then the circular arcs will coincide with the soffit. For bridges of small obliquity two circular arcs will suffice, and the template would in other respects be simplified.

In Fig. 13, A B is the same fraction of the soffit semicircumference that A D, B C at right angles to A B are of the axial lengths of the joint beds and facial screw lines. On B D, as base, erect the parallelogram *k h e f*, the sides *k h*, *f e* being parallel to C A, and equal in length to the thickness of a voussoir; the sides *k f*, *h e* are equal to the length of a voussoir: through *k* draw *k g* parallel to either A D or B C. If a piece of sheet iron about $\frac{1}{16}''$ thick be cut out similar and equal to the figure *k f e h*, it will be the same figure as the development of the soffit of a voussoir; if now divisions of feet and inches be marked on the sides *f k*, *e h*, and the plate be made to coincide throughout with the soffit of a voussoir already dressed, we may then use it for marking off the soffit of a voussoir, of which the soffit and one bed only have been dressed, the other soffit joint line and the two cross lines; we can by it also measure different lengths of voussoirs accurately. If the figure *k g h* be treated in a similar manner, it is manifest that when *k h h g* coincide with a cross line and a joint line, *k g* will be parallel to the axis of the cylinder.

Fig. 14 consists of three arms, A B, B C, C A, of plate iron about $\frac{1}{16}''$ thick and 1'' wide. The rods C A, C B are attached to the rod A B by the joints *e d*, so placed that the edges *a*, *b*, *c* pass through the centres of the pins of the joints; the distance between the centres of *e* and *d* is a certain fraction of the radius of the soffit, and the divisions of feet and inches on the rods C B, C A, reckoning from the centres of the axes *d*, *e*, will enable us to mark off on the edges *c*, *b* the same fractions of the distance of the point of intersection of any joint line with the facial ellipse from the centre of radiation, and of the centre of radiation from the foot of the corresponding radius, respectively. E is a clamp with screw attached, suspended from the end of the rod B C, by which the rods B C, A C may be clamped in any position. A B E D C, Fig. 15, is a section of the springing, dressed for the reception of the voussoirs, taken at right angles to the face, and in such a position that A C is equal to the projection of the soffit thickness of a voussoir on an arc of the soffit; C D, A E, are radial lines; the figure E A B is a similar section of the springing of a direct bridge of the same span and rise of arch; templates similar to these two figures must be cut out in sheet iron or wood.

IV. Method of using the templates.

Template, Fig. 11*b*, is used for dressing off the first bed of a voussoir. Chisel drafts are sunk in the stone, so that the edges B D, B′ E of the template coincide with them, when the template is applied to the stone; the bed is then dressed off, so that a straight edge will move freely over it in contact with the drafts, when it is kept at right angles to them.

Template, Fig. 12*a*, must now be used. Chisel drafts must be sunk on the soffit, so that when the edges *h e*′, *k f* ′ coincide with the bed, the circular arcs may coincide with the soffit. The soffit must then be dressed off, so that a straight edge held at right angles to these drafts will move freely over in contact with two or more of the drafts. By one operation, as we see from Figure 12*a*, the dotted lines may be taken to represent the joint lines; only the middle part of the soffit of the voussoir can be dressed, but the portions left towards A and *d*, if the length A F H be about the length of a voussoir, and the obliquity be not very great, or if the radius be so large that there is not much twist in any single voussoir bed, can readily be dressed off so as to harmonise with the rest. If it be necessary, more circular drafts may be sunk, care being taken that one of the arcs coincides with the part already dressed.

This soffit may now be used for giving the template, Fig. 13, the proper twist, and the template then used for marking on the soffit the remaining joint lines and the two cross joint lines. Template, Fig. 12*a*, must now be applied, and drafts sunk on the other bed passing through the joint line marked on the soffit, so that the edges of the rods may coincide with the drafts, whilst the arcs coincide with the soffit. This second bed must be dressed off with a straight edge in the same manner as the first. Radial lines must now be marked on the beds, passing through the intersections of the lines marked on the soffit, either with template Fig. 11*b* or template Fig. 12*a*, and the ends of the voussoir dressed off by these lines. The voussoir thus dressed will be an ordinary voussoir; the ends are like the beds' screw surfaces: every voussoir, whether on the springing or on the face, will have one end similar to these, except two or three at the obtuse angle. In Fig. 20, which is a development of the soffit, the figures to the right of the development of the ellipse line A E′ show the lengths of the facial voussoirs along the face A E′: their lengths along the face C E will be the same, commencing at C; similarly the figures to the left of the development B E show these lengths along the face B E, which are the same as those along D E′ commencing at D. The two soffit joint lines of each facial voussoir are taken to be of the same length; they will practically be so whenever the rise of the arch is so small that, as in the present case, the facial ellipses differ very little from the facial screw lines. The dimensions in the two columns to the left of the development A E′, opposite the 1st, 2nd, &c., face joint, are the lengths of the arms of the template, Fig. 14, corresponding to those joints; they will practically be the same for the joints along B E. On comparing these dimensions, we see that the divisions of feet and inches on the longer arm may begin with 3′ 6″, and on the shorter with 1′ 6″. Suppose we fix upon any joint, the 15th, for which the template is set, the two beds, soffit, and one end of the voussoir having been already dressed, the length of the voussoir must be marked on each soffit joint line, and a line drawn on the soffit connecting them. The mason must now mark a radial line on the bed facing him, passing through the facial extremity of the voussoir; template, Fig. 15, must then be applied, so that the edge *a* corresponds with the radial line, and the axis of the joint *d* is on the soffit joint line; the edge *c* will be in direction of a joint line of the face. A line must now be marked on the bed along the edge *c*, and the end of the voussoir dressed off flush with this line and the line previously marked on the soffit.

The mason is supposed to commence at the springings A, B, consequently the dressed ends of the facial voussoirs will be to his right, along the face commencing with the acute angle, and to his left along the face commencing at the obtuse angle. The obliquity of the bridge, chosen for illustration, is so great that the portion B E is equal to half the length of the elliptic arc, within a fraction of a voussoir thickness. When the bridge is of less obliquity, there will be facial voussoirs on both sides of the centre in the part A E. In passing the centre, the radial line and face line correspond; and for voussoirs beyond the centre, the edge *c* will be on the contrary side of the edge *a*; but if applied to the opposite bed, so that the dressed end of the voussoir is to the left, the edge *c* will be on the same side of the edge *a*, as before. Since the voussoirs on each side of the centre on one face will be the same as those on the corresponding sides of the centre on the other face, the workman ought to dress the facial voussoirs only as far as the centre of each face, commencing from each angle of the springing. Two of each ought to be dressed and marked, so that no mistakes may arise. In the example the acute angle is to the left; if it had been to the right, the graduated arms, Fig. 14, would have to be clamped to the left of *a*. In this case the edge *a* and the joints *d e* ought to be on the other side of the arm A B. In Fig. 20 the springing is dressed off, as if for a direct bridge. The springing stones must be thus dressed. The length of the longest joint line, which is here equal that of an ordinary voussoir, or 3′, must be marked on the under soffit joint line, the stones being so placed, relatively to the workman, that the soffit is towards him, and the dressed end to his right. Template, *k h g*, Fig. 13, must now be applied, so that the point *g* coincides with the extremity of the longer soffit joint line, whilst *h g* coincides with that line throughout. A line marked along the edge *g k*, on the soffit will be parallel to the axis. Radial lines must now be marked on the beds, passing through the points of intersection of this soffit line with the joint lines, and the end dressed off flush with these lines. At the obtuse springing there are three voussoirs, none of whose faces are screw surfaces. The springing must be dressed off first, and the soffit lengths taken from the plan marked on each bed; template, Fig. 14, must then be applied, as before.

If the springing is to be dressed off ready for the reception of ordinary voussoirs, the springing stones, before being set, must be dressed, so that B A C D, Fig. 15, represents a section of them taken at right angles to the face of the abutments; towards B E D, the outline may present any shape whatever, but the section must not be less than the figure A B E D C—A B being horizontal. The springing so dressed will be that represented in Fig. 16*a*. A B is the horizontal line of the springing; this must be divided into a number of parts, A D, D F, F H, &c., each equal to the length of springing intercepted between two contiguous soffit voussoirs. Template, Fig. 13, *h k g*, must now be applied, so that the side *k g* corresponds successively with A D, D F, F H, &c., and the joint and end lines, A *r*, *r* D, D *m*, *m* F, &c., marked on the soffit; if, now, lines be marked on the upper bed through *r*, *m*, *n*, &c., at right angles to the line O E, they will be in direction of the revolving radius. Drafts must now be sunk above D, F, H, &c., at right angles to D E or A B, until they coincide with radial lines passing through D, F, H, &c.: if then the beds D M, F N, &c., and the ends M F, N H, &c., be dressed off by these drafts, the springing will be represented by Fig. 16*b*. The first radial line next the acute angle A, so great is the obliquity, will cut the oblique face of the springing, and the first voussoir, resting on the bed *e c*, will project beyond the faces.

Fig. 17 is an elevation of this face, *o*, *o′* being the centres of the cylinders and of the radiation of the face joints. When the springing is dressed off to receive the voussoirs, A B, C D will be the springing joint lines; and when the projection above mentioned is dressed off, A *p b* will represent the visible portion of that voussoir. At the oblique angle of the springing, the depth, B *s*, is less than the thickness of a voussoir, we must, therefore, sink drafts through *s* and *t* until they coincide with radial lines; the point, C, the extreme point on the extrado, in Figs. 16*a*, 16*b*, on account of the obliquity, will be higher than the rest of the extradosal points on the springing; that springing stone must, therefore, be of extra depth on the extrado. The bed must be dressed off, so that a straight edge, held parallel to the face, will move freely over the surface, in contact with the drafts t_n, *s r*.

When the springing is dressed off, as if for a direct bridge, the lines F B, C E, Fig. 17, will be the first facial lines on the springing; at the acute angle, the first voussoir specially dressed for the springing will present at the face of the arch the figure A F B: at the obtuse angle, the soffit thickness of the first is less than that of an ordinary voussoir; but on the extrado it is increased, since the face, instead of presenting the figure M N D C, will present the figure M N D E C; now the figure D E C is exactly equal and similar to the figure A F B, and the portion cut off from the voussoir at the acute springing will be exactly equal to the portion required at the obtuse.

In bridges of great obliquity, the lengths of the springing beds, D *m*, F *n*, Fig. 16*a*, are so short, that template, Figs. 11*a*, 11*b*, cannot be used. Fig. 19 represents a springing dressed off for an an angle of 15° obliquity, the rest of the data being the same; the soffit lines, D *m*, *m* F, F *n*, *n* H, &c., being marked upon this, the lines *m* M, *n* N, at right angles to the face, will be radial lines—the soffit being previously dressed as in Fig. 16*a*; template, Fig. 11*b*, may now be used, one edge coinciding with the radial lines *n* N, *m* M, &c., and a draft sunk, so that the other coincides with it, and meets a joint line; the bed must then be dressed off by these drafts.

It thus appears that, in bridges of great obliquity, it will be best to dress the voussoirs to fit the springing, and, when of small obliquity, to dress the springing to fit the voussoirs.

THE END.

LONDON:
PRINTED BY C. WHITING, BEAUFORT HOUSE, STRAND.

FIG. 1

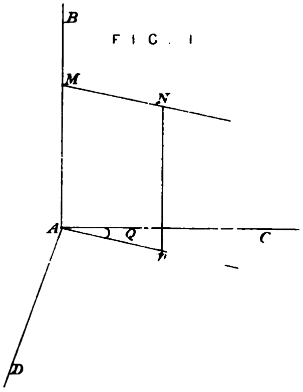
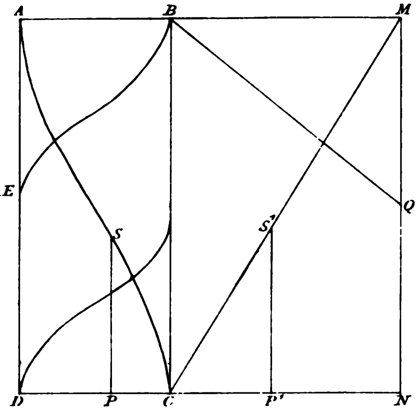


FIG. 2



C 3

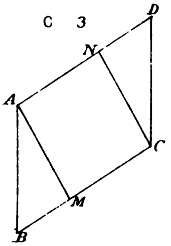


FIG. 5

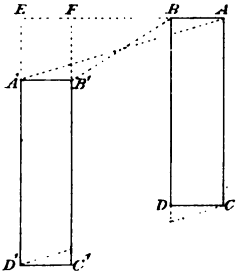


FIG. 4

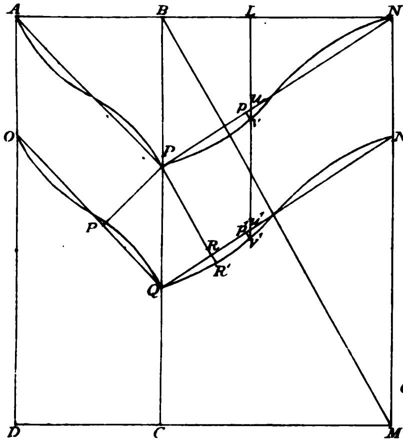
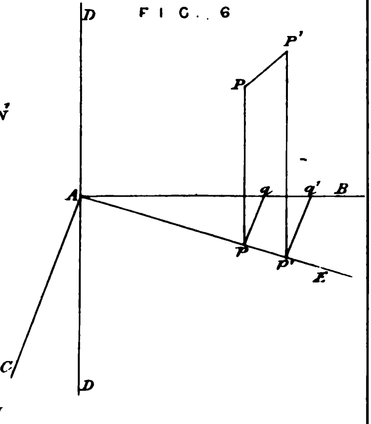


FIG. 6



Newberry & Alexander, Lith. 43, Castle St Holborn

E. & F. N. Spon. 48. Charing Cross. London.

FIG. 7.

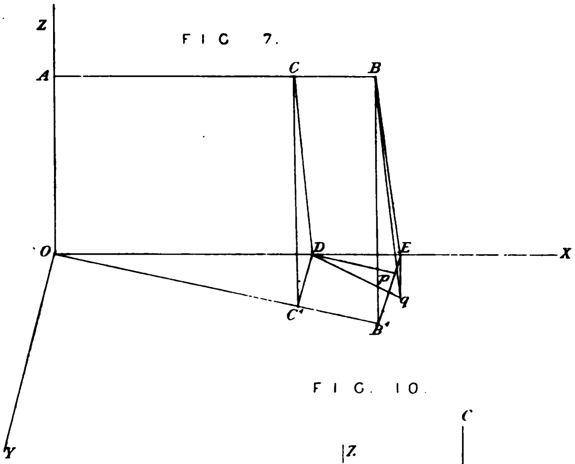


FIG. 10.

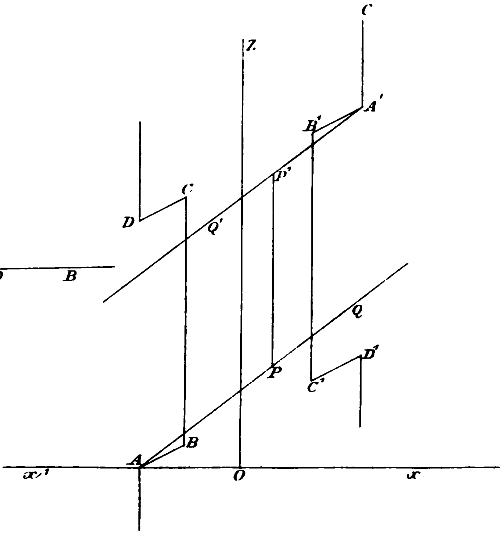


FIG. 8.

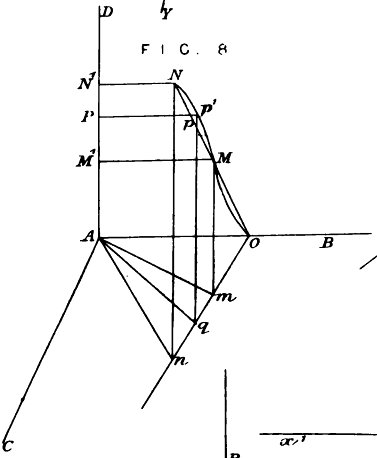
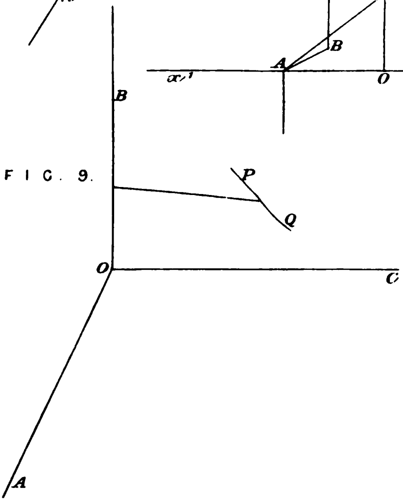


FIG. 9.



E.&F.N Spon.48,Charing Cross,London.

FIG 11^a

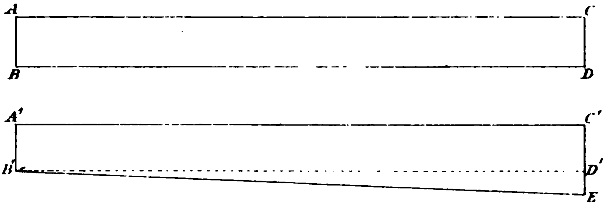


FIG 11^b

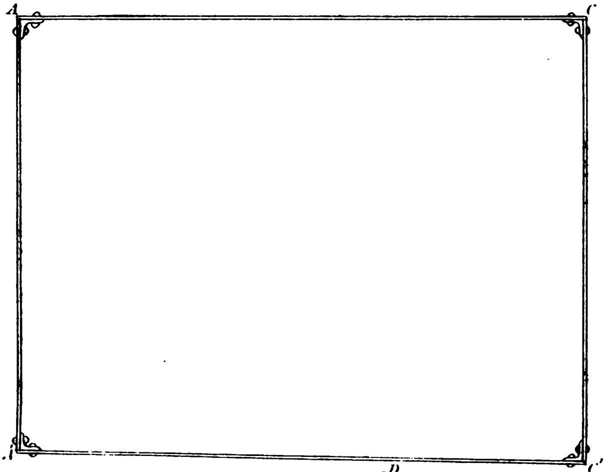
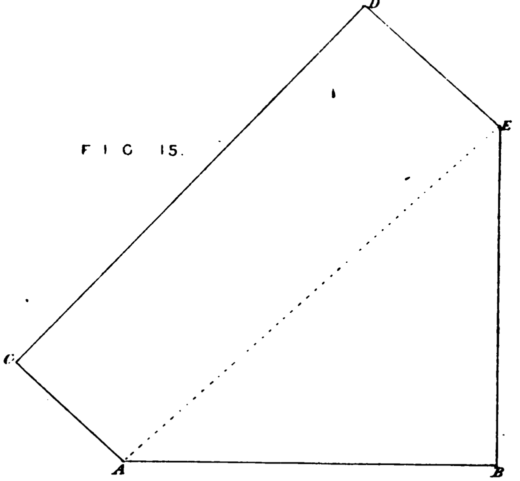


FIG 15.



E.&F.N. Spon. 48. Charing Cross. London.

Plate 4

FIG. 12^a

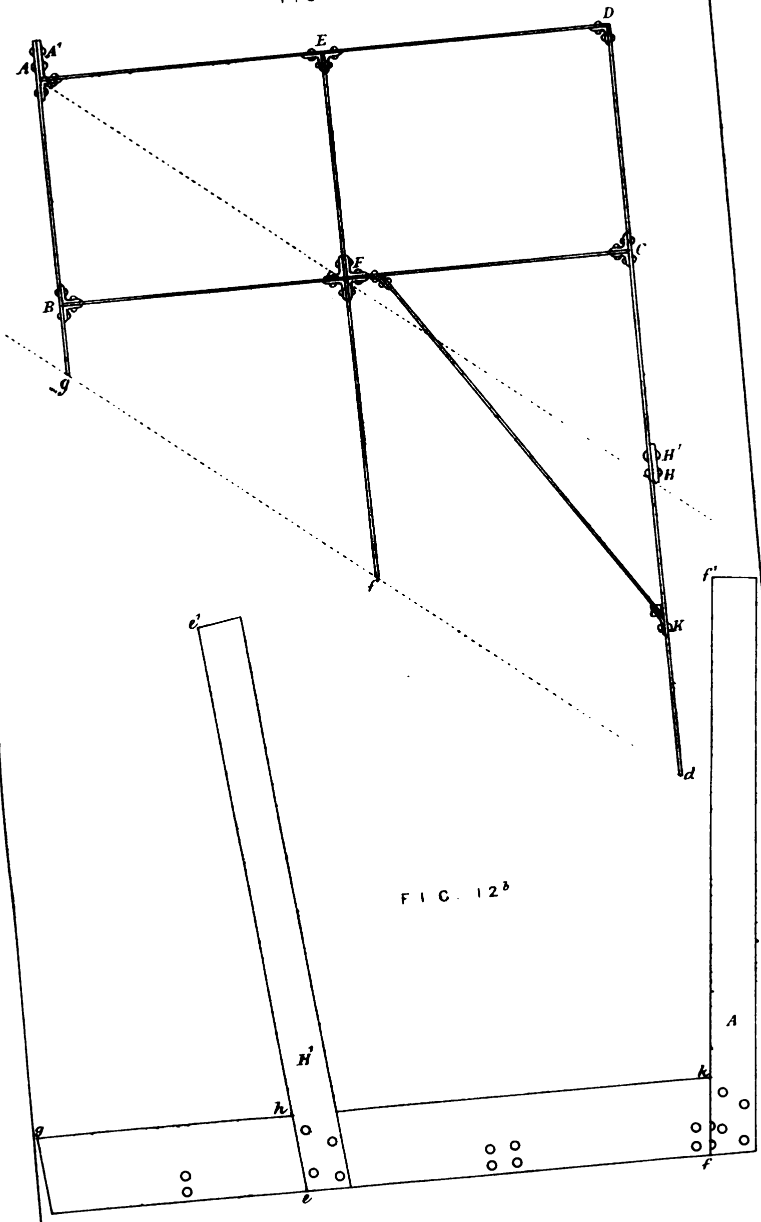
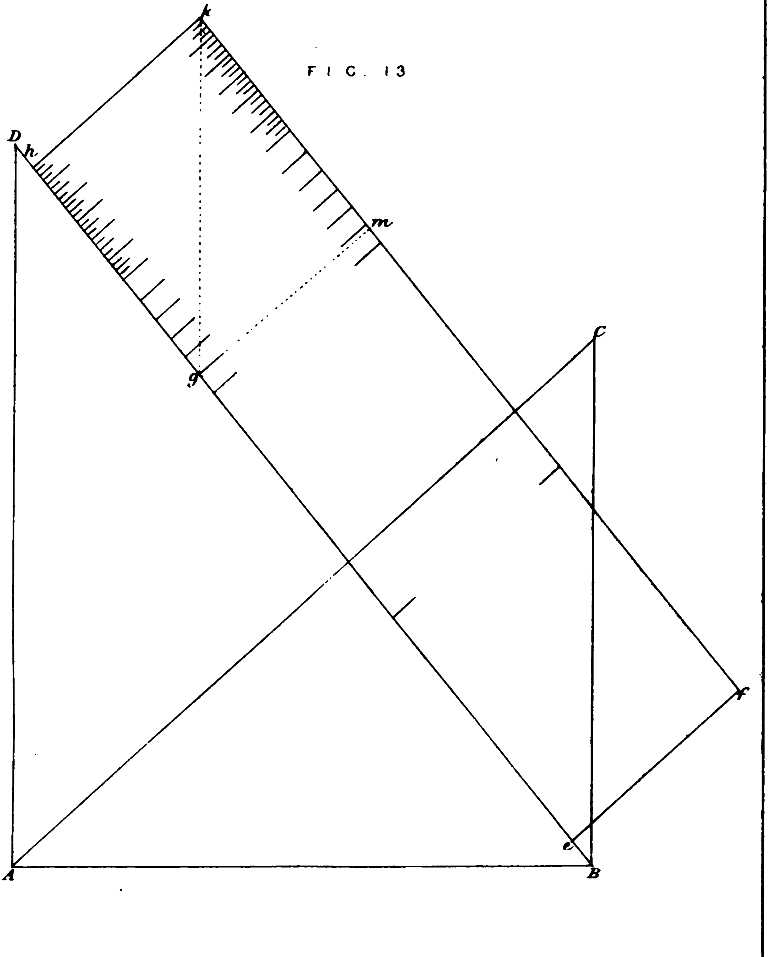


FIG. 12^b

E. & F. N. Spon. 48, Charing Cross. London.

FIG. 13



E. & F. N. Spon. 48. Charing Cross. London.

Plate 6

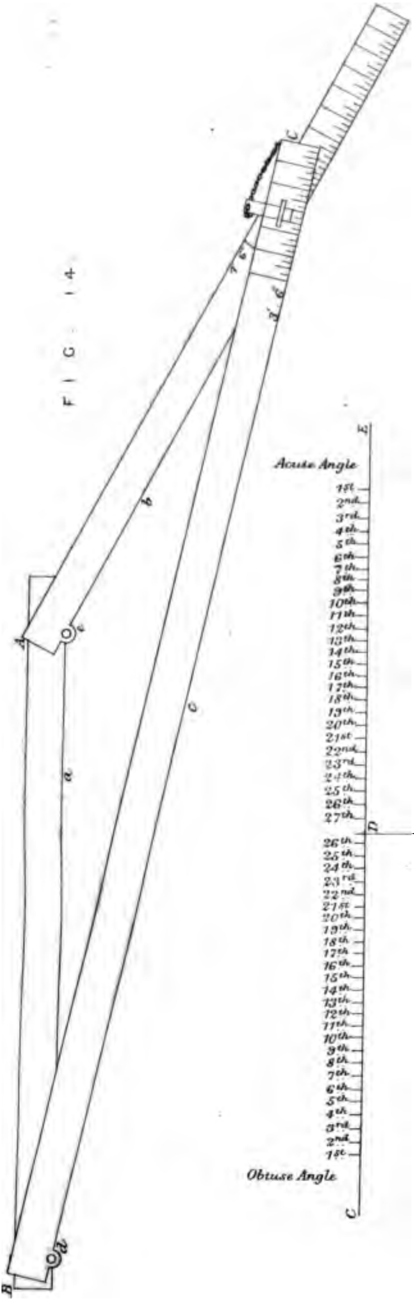


FIG. 14.

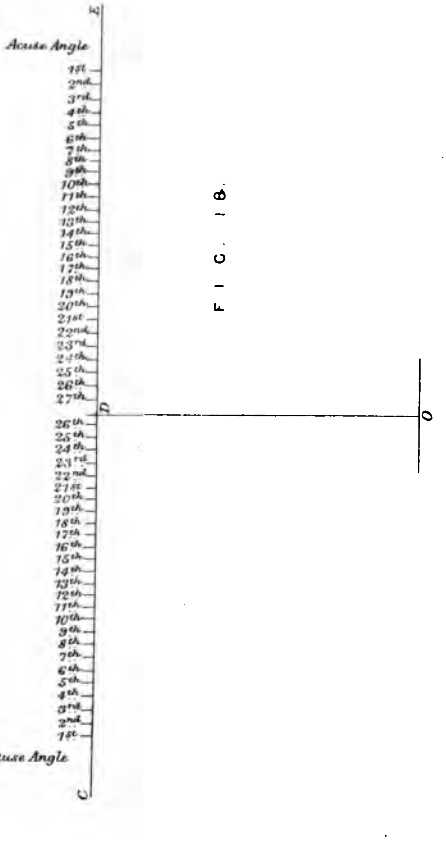


FIG. 18.

E. & F. N. Spon, 48, Charing Cross, London.

FIG. 16^a

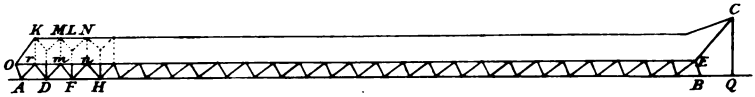


FIG. 16^b

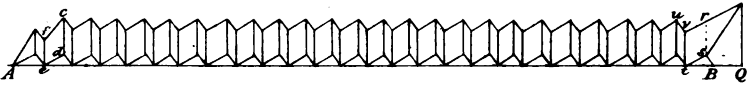


FIG. 17

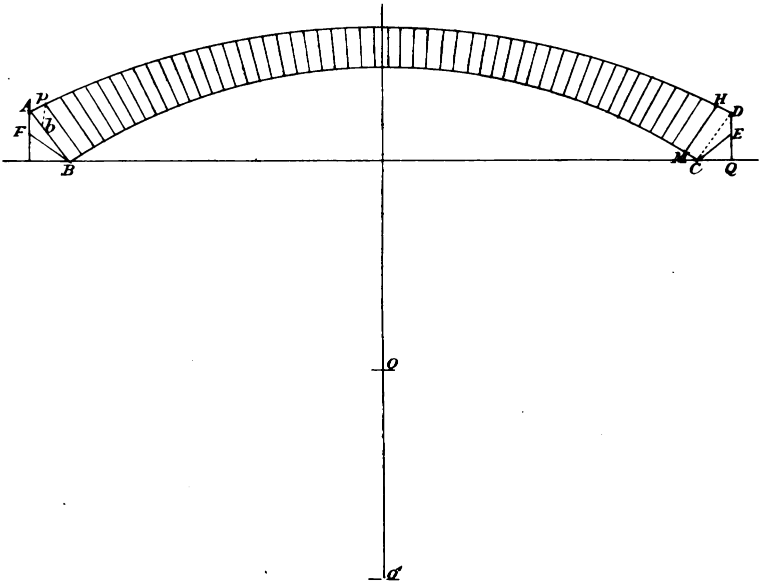
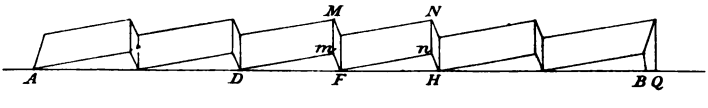


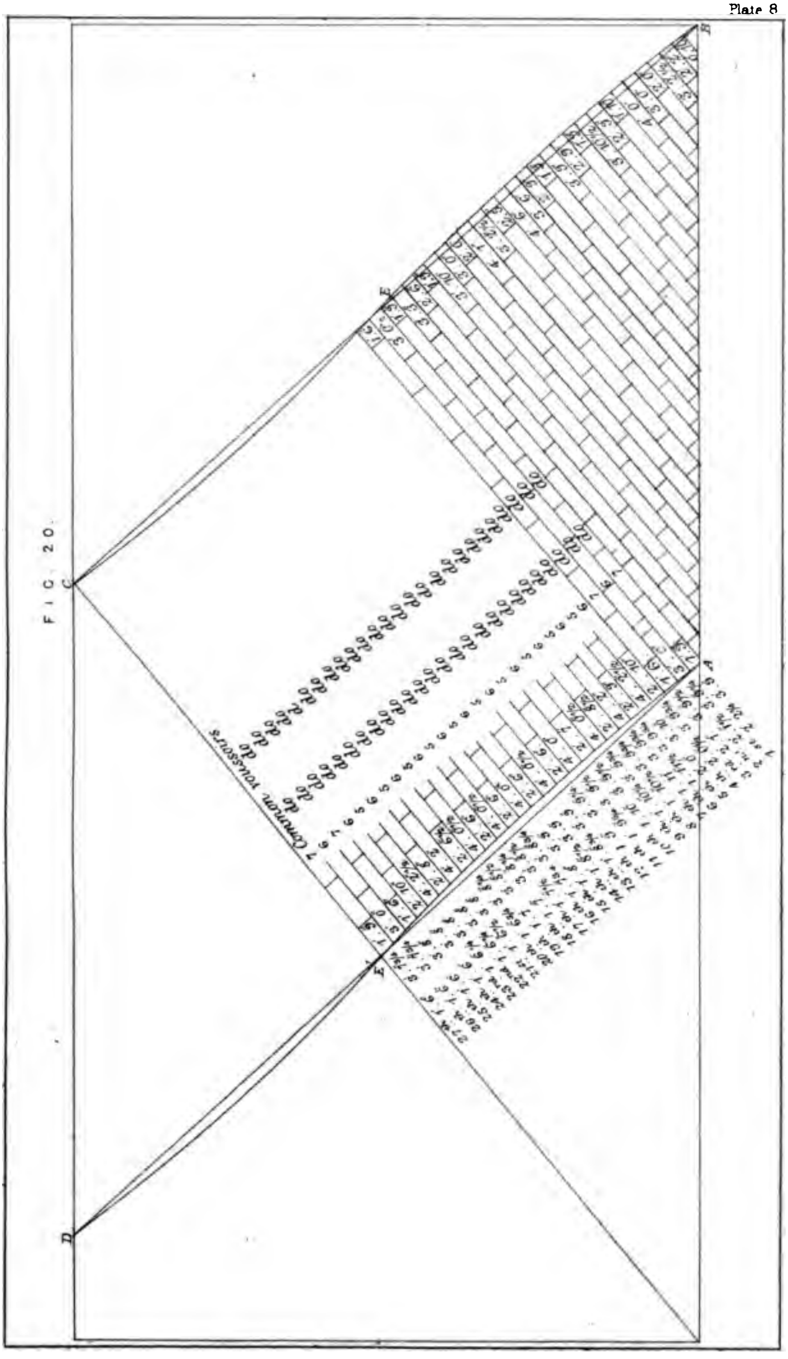
FIG. 19.



E.&F.N. Spon, 48, Charing Cross, London.

Plate 8

FIG. 20.



E & F. N. Spon. 48. Charing Cross. London.